



Association Analysis of Microbiome Presence-Absence Data using Logic Regression

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Joint work with Yiwen Chen

Prominence of Microbiome Research

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Logic Regression

Bottom-Up Procedure

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Differential Abundance Analysis

Censoring Transformation

Log-Rank Test

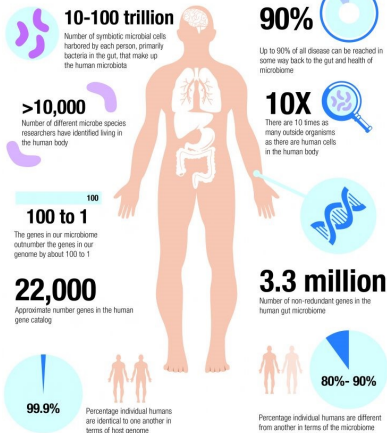
Summary

- Numerous associations with health conditions
- Promising therapeutic innovations
- Potential causal effects on disease
- A burgeoning field with lots of unknowns



The Importance of the MICROBIOME

By the Numbers



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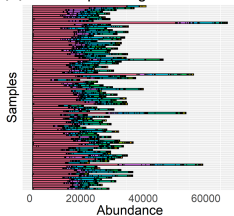
Differential Abundance Analysis

Censoring Transformation

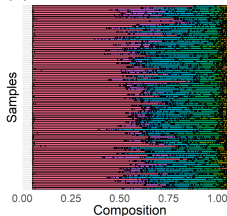
Log-Rank Test

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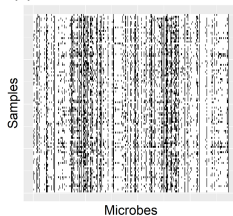
(a) Sequencing Reads



(b) Relative Abundances



(c) Presence-Absence Data



- Read counts from amplicon or metagenomic sequencing data
 - Heterogeneous sequencing depths

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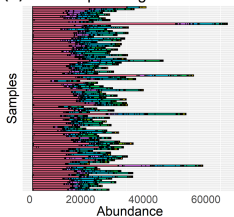
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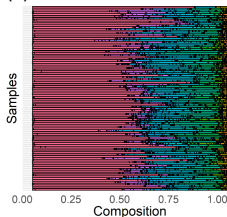
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Summary

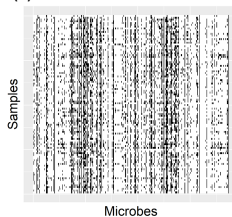
(a) Sequencing Reads



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(c) Presence-Absence Data



- Read counts from amplicon or metagenomic sequencing data
- Relative abundances after normalization
 - Compositionality; skewness; zero-inflation

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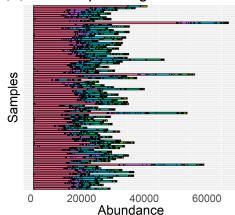
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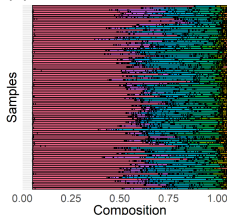
Log-Rank Test

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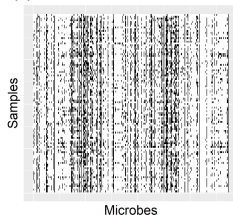
(a) Sequencing Reads



(b) Relative Abundances



(c) Presence-Absence Data



- Read counts from amplicon or metagenomic sequencing data
- Relative abundances after normalization
- Dichotomized presence/absence (P/A) states

Statistical Challenges

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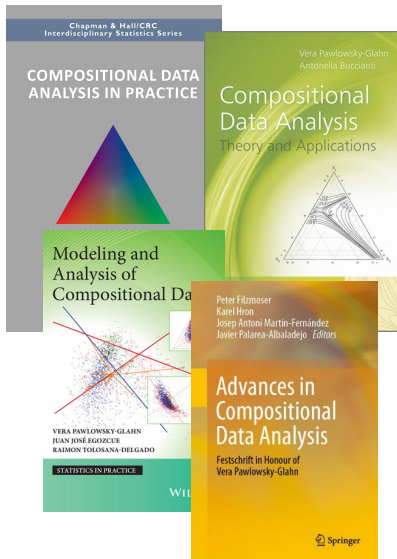
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Summary

- **Compositional data analysis is tricky**
 - ★ Non-Euclidean
- **Highly skewed**
 - ★ Dominant vs. rare taxa
- **Excessive zeros**
 - ★ Typically over 50%
- **Tree structure**
 - ★ Taxonomy or phylogeny
- **Measurement errors**
 - ★ Esp. at finer levels



Benefits of P/A Analysis

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- P/A states of taxa have intrinsic health implications
 - E.g., presence of *E. coli* causes UTI
- Binary data are much easier to analyze
- More robust against measurement errors
- Better suited for rare taxa analysis
- *When conducted tactically*, P/A dichotomization may preserve almost all the abundance information*



Fundamental Tasks in Microbiome Research

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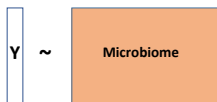
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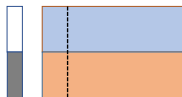
Censoring Transformation
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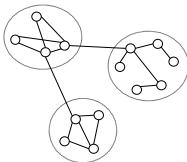
■ Regression/Association Analysis



■ Differential Abundance Analysis



■ Network Inference



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Regression Analysis

Novel Tree-Guided Logic Regression for P/A Data



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Log-contrast models

$$Y = \beta_0 + \beta_1 \log(X_1/X_p) + \cdots + \beta_{p-1} \log(X_{p-1}/X_p) + \varepsilon$$

or

$$Y = \beta_0 + \sum_{j=1}^p \beta_j \log X_j + \varepsilon, \quad \text{where } \sum_{j=1}^p \beta_j = 0$$

- (X_1, \dots, X_p) is the vector of relative abundances
- Inadequate to handle zeros
- Lacking straightforward biological interpretations
- Subject to measurement errors

P/A Dichotomized Predictors

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$$Y = \beta_0 + \sum_{j=1}^p \beta_j X_j + \sum_{k=1}^q \theta_k L_k + \varepsilon,$$

- $X_j \in \{0, 1\}$ is the P/A status of Taxon j
- L_k is a logic expression (Boolean operation of X_1, \dots, X_p)
 - Regular interaction (e.g., $L_k = X_j X_{j'}$)
 - ✓ Pros: existing methods available (e.g., quadratic regression)
 - × Cons: undesirable interpretation; heredity constraint
 - Arbitrary logic expression (e.g., $L_k = X_1 \vee X_2 \vee X_3 \wedge X_4^c$)
 - ✓ Pros: flexible; logic regression ([Ruczinski et al., 2003](#), JCGS)
 - × Cons: obscure biological meaning; slow to fit

Tree-Guided Logic Expression

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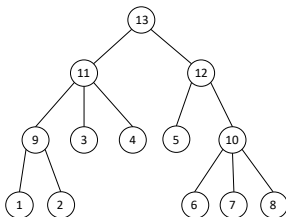
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Summary



- Phylogenetic tree or taxonomic tree
- Leaf nodes: X_1, \dots, X_p
- Internal nodes: L_1, \dots, L_q
- Only use *OR* to combine descendant leaf nodes; for example,
 - $L_9 = X_1 \vee X_2$
 - $L_{12} = X_5 \vee X_6 \vee X_7 \vee X_8$

Model Fitting

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$$\min \|Y - \mathbf{X}\beta - \mathbf{L}\theta\|^2 + \mathcal{P}_\lambda \left((\beta^T, \theta^T)^T \right)$$

- $\mathcal{P}_\lambda(\cdot)$ is a sparsity-inducing penalty (e.g., LASSO)
 - ✓ Pros: ready to implement
 - × Cons: collinearity; overlapping features
 - ★ **Proposal: a bottom-up combination/selection procedure**



Bottom-Up Procedure

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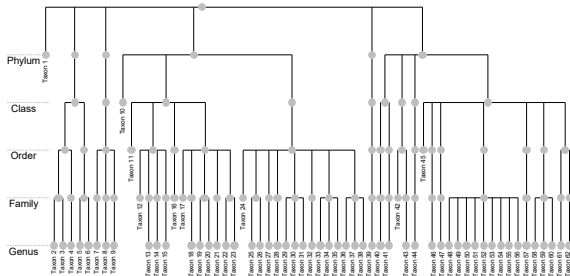
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Summary

- A greedy heuristic algorithm
- First, **combine** eligible nodes that lead to the steepest decrease in BIC (tradeoff btw *goodness-of-fit* and *parsimony*)
- Once BIC stops decreasing, further **select** features



Bottom-Up Procedure

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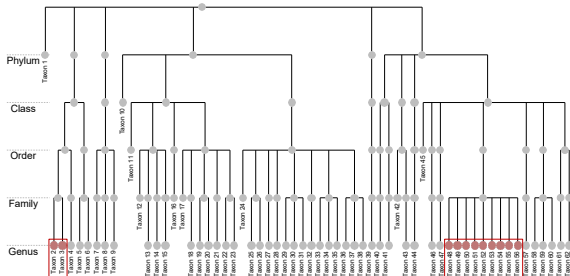
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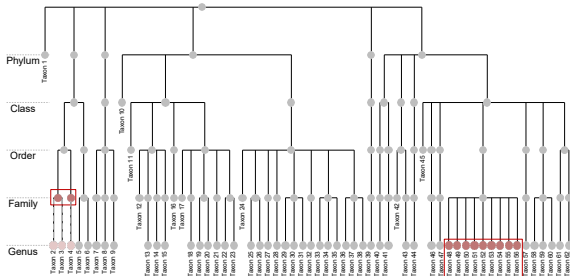
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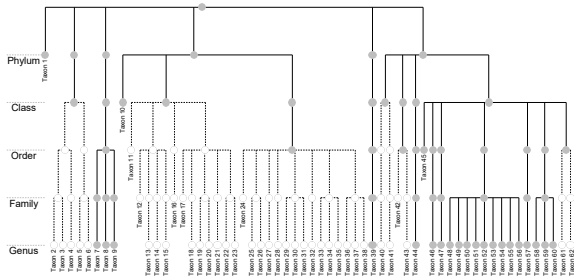
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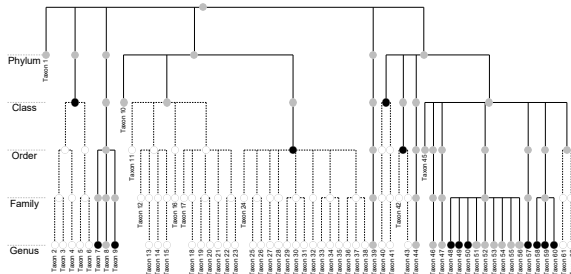
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Properties

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Summary

Compared to naive variable selection methods (either for leaf nodes only or for all nodes), the BU method

- Accommodates the tree structure
- Better alleviates the collinearity
- Selects fewer variables
- Has better interpretability



Simulation

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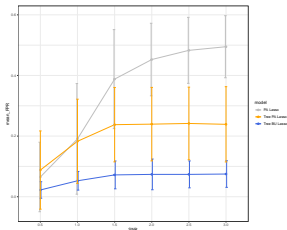
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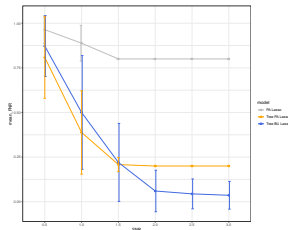
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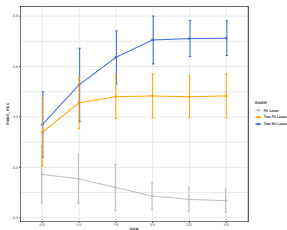
Summary



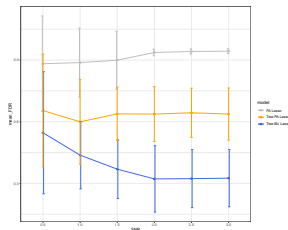
(a) FPR



(b) FNR



(c) MCC



(d) FDR



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Summary

Oral Infections, Glucose Intolerance and Insulin Resistance Study (ORIGINS)

- Goal: associate oral microbiota with periodontal health
- 757 diabetes-free individuals (WAVE II)
- 16S rRNA sequencing on subgingival plaque samples
- 530 taxa at the OTU level with known taxonomic structure
- Periodontal status (percent bleeding on probing) as outcome
- Sex, age, BMI as covariates



Evaluation Metric

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- Randomly selected 500 samples for training and 257 for testing
- Repeated 50 times
- Compared BU (proposed), LASSO (leaf nodes only), Tree-LASSO (all nodes)
- Each time, evaluated the following
 - In-sample MSE (goodness of fit)
 - Out-sample MSE (prediction performance)
 - Selected features (interpretability)



Results: MSE

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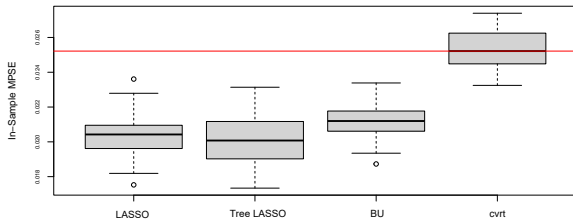
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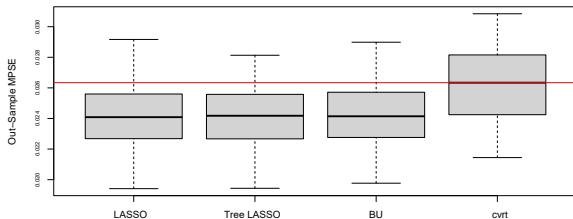
Summary



Model Fitting Comparison



Prediction Comparison



Results: Feature Selection

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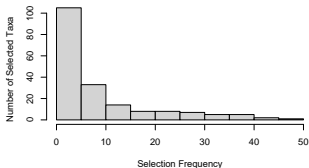
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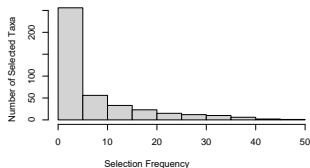
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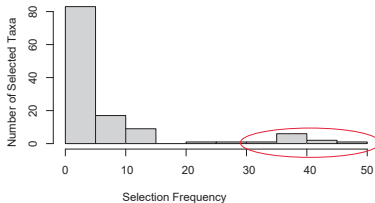
LASSO



Tree-LASSO



BU (Proposed)



Results: Feature Selection

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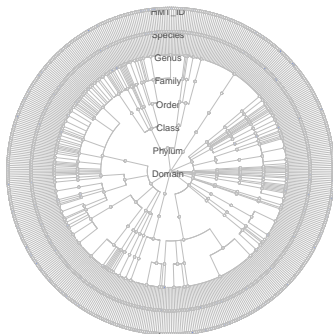
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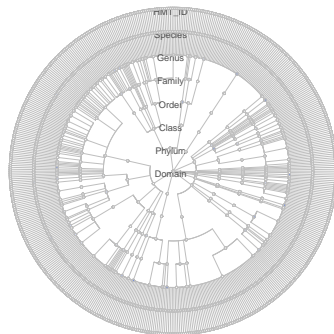
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(a) Tree-Lasso



(b) BU

- BU selection is much more sparse and stable

Results: Feature Selection

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Top Selected Taxa by BU



- Top selected taxa (>40%) at different taxonomic levels by BU
- Their P/A is associated with periodontal status
- The evidence in literature corroborates with our findings

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Novel Transformation and Censored Data Analysis



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Goal: identify differentially abundant taxa in different groups

- Parametric tests rely on zero-replacement transformations
- Nonparametric tests are not good at handling ties
- Inadequate covariate adjustment

Is there a better way to handle “0”?

Where is “0” from

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Summary

- Structural zeros
 - “True” zeros
 - Absence of a taxon in a sample
- Sampling zeros
 - “Pseudo” zeros
 - Fail to detect the existence due to low abundance or insufficient sequencing depth



Where is “0” from

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 - Fail to detect the existence due to low abundance or insufficient sequencing depth
- Both are due to **actual abundance below detection limit**



Where is “0” from

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 - “True” zeros
 - Absence of a taxon in a sample
- Sampling zeros
 - “Pseudo” zeros
 - Fail to detect the existence due to low abundance or insufficient sequencing depth
- Both are due to **actual abundance below detection limit**

That's **censoring!**



Treat "0" as Censored

Assume detection limit is 1

Samples	OTU					Library Size
x_1	19	1	78	0	0	$m_1=98$
x_2	5	0	41	2	0	$m_2=48$
↓						
x_1^*	19	1	78	1 ⁻	1 ⁻	$m_1^*=100$
x_2^*	5	1 ⁻	41	2	1 ⁻	$m_2^*=50$
↓						
$C(x_1^*)$	0.19	0.01	0.78	0.01 ⁻	0.01 ⁻	
$C(x_2^*)$	0.10	0.02 ⁻	0.82	0.04	0.02 ⁻	
↓						
z_1	$-\log(0.19)$	$-\log(0.01)$	$-\log(0.78)$	$\{-\log(0.01)\}^+$	$\{-\log(0.01)\}^+$	
z_2	$-\log(0.10)$	$\{-\log(0.02)\}^+$	$-\log(0.82)$	$-\log(0.04)$	$\{-\log(0.02)\}^+$	

Interpretation

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	Microbiome Data	Survival Data
Type	Right-censored	Right-censored
Range	$[0, \infty)$	$[0, \infty)$
Time	Abundance cutoff (high to low)	Time duration (short to long)
Event	Presence	Death
Censoring	Zero count	Dropout
At-risk	Abundance at or below (P/A!)	Survival time at or above



Visualization (for one taxon)

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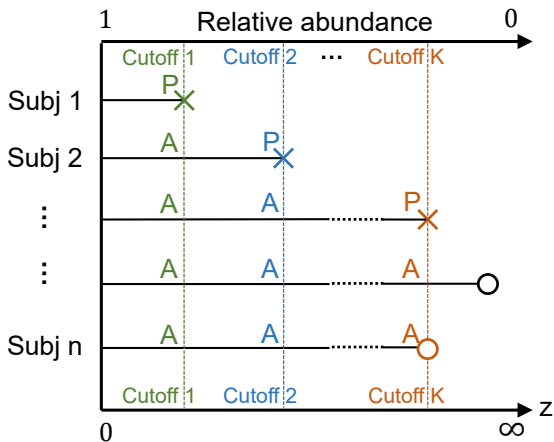
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For each taxon, test for equality of distribution

$$(z_1^{(1)}, \dots, z_{n_1}^{(1)}) \text{ vs. } (z_1^{(2)}, \dots, z_{n_2}^{(2)})$$

- Classic two-sample test in survival analysis
 - Without covariate: log-rank test
 - With covariates: Cox model
- No distributional assumption on z



Log-Rank Test (for one taxon)

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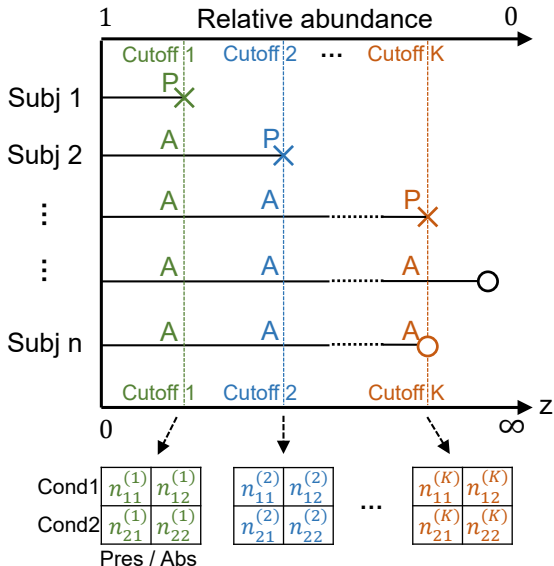
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Additional Remarks

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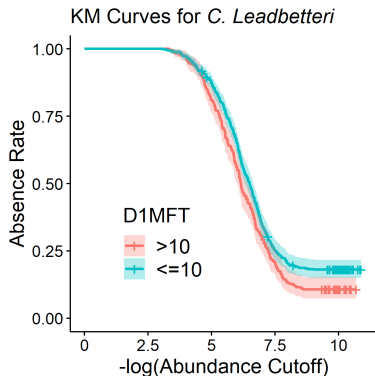
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- **Better zero handling** (by aggregating P/A information across different cutoffs)
- **More powerful** in detecting differences at lower abundance levels (suitable for rare taxa comparison)
- **Highly flexible** (different variants available; Log-rank test is equivalent to the score test in Cox)



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P/A-based methods have untapped potential for microbiome studies

- Easier to analyze
- Less sensitive to measurement error
- Better suited for rare taxa
- New method developments
 - Interpretable regression analysis
 - Differential abundance analysis
 - Co-occurrence network inference

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Thank you!

*Also working on methods for longitudinal microbiome data.
Interested to know more?*

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